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ON A CLASS OF GOAL PROGRAMMING MODELS FOR A SINGLE FIRM.(U)
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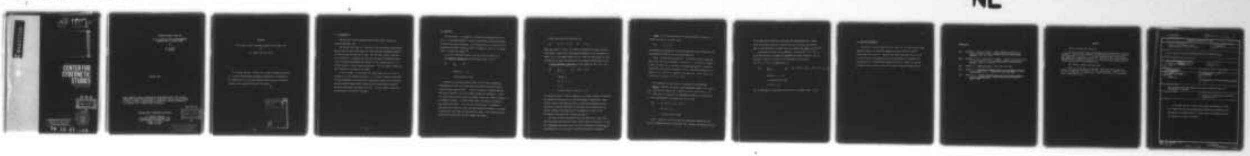

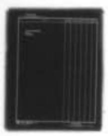

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Research Report CCS 353

ON A CLASS OF GOAL PROGRAMMING MODELS FOR A SINGLE FIRM

by

A. Charnes
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October 1979

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ABSTRACT

On a Class of Goal Programming Models for a Single Firm

by

A. Charnes and Sten Thore

It is shown that for a broad class of goal programming problems for a single firm and goal programming formulation is identical to the imposition of a fictitious system of excise taxes or subsidies levied on the outputs and inputs of the firm.

1. Introduction.

The methods of goal programming have recently been surveyed by Charnes and Cooper [1].

Our present work draws its inspiration from the doctoral dissertation by Ryan [5], who discovered that in a particular goal programming model of the distribution and costing of labor, land and other primary resources, it was possible to establish the solution, and thus to simulate the workings of the goal programming formulation, by the introduction of a fictitious system of excise taxes and subsidies. The unit taxes or subsidies were obtained as the duals of the goals.

It will be shown in the present note that these results are but an instance of a quite general principle. It is true for a quite broad class of goal programming formulations that the goaling feature can be replaced by an alternative hypothetical system of excise taxes and/or subsidies levied on the outputs and inputs of a firm. The unit taxes or subsidies are obtained as the duals of the goals.

2. Analysis.

Let the vector y, of dimension n, denote the production plan of a firm. In the common fashion[1], outputs are entered as positive elements, and inputs as negative elements. The production set (i.e. the set of all technically possible production plans) is denoted Y, with $y \in Y$. Assume that Y is convex and compact.[2]

Let p be a nonnegative vector of prices, also of dimension n.

The classical formulation of the problem of the firm is:

$$(1) \qquad \max_{y} \quad p^T y$$

$$\text{subject to} \quad y \in Y$$

$$y \text{ unrestricted in sign}$$

We shall in the present paper study a class of goal programming formulations for this firm, where goals have been laid down for one or several outputs and/or inputs. In order to provide a suitable notation for such a situation, let d be a vector of goals of dimension n. Goals for outputs are entered as positive elements, goals for inputs are entered as negative elements. (In other words, goals are formally attached to all outputs and inputs. If in the real world no goal is attached to some particular output or input, this may formally be handled in the model by putting the goal equal to some arbitrary number, and allowing for zero penalties for deviations from this number; see below.)

Define deviations from the goals by

(2) $$y + y^{+} - y^{-} = d, \qquad y^{+}, \quad y^{-} \geq 0$$

where the vectors y^{+} and y^{-} are vectors of under-fulfillments and over-fulfillments, respectively (the goaling procedures to be introduced in a moment will in the common manner see to it that for each output or input at least one of these two deviations will always be driven down to zero).

A <u>goal-programming formulation</u> of the problem of the firm is then

(3) $$\max_{y,\, y^{+},\, y^{-}} \quad p^{T}y - c^{+T}y^{+} - c^{-T}y^{-}$$

subject to

$$y + y^{+} - y^{-} = d$$

$$y \in Y$$

$$y \text{ unrestricted in sign, } y^{+},\ y^{-} \geq 0$$

The vectors c^{+} and c^{-}, both of dimension n, are known vectors of penalties for under-fulfillments and for over-fulfillments, respectively. They reflect either actual monetary costs or perceived psychological costs. They may be cardinal magnitudes, or they may only be specified as an ordinal ranking of the importance of the goals by the use of non-Archimedean branscendentals ("preemptive goals").

The class of goal programming models (3) embraces all cases where goals have been laid down for outputs and/or inputs of the firm. It does not accommodate cases where goals have been formulated for parameters of the production set (e.g. goals on technical productivity measures).[3)]

Lemma. It is always possible to find some vector M of positive numbers, so that if the constraints

(4) $y^+, y^- \leq M$

are adjoined to problem (3), the new and more restricted problem has the same optimum solution as the original problem (3).

Proof: The difference vector $|y^+ - y^-|$ cannot exceed in magnitude some vector M of positive numbers, since $y^+ - y^- = d - y$ and Y is compact. Further, we know that at the point of optimum each goal will either be under-fulfilled (a positive element of y^+) or over-fulfilled (a positive element of y^-); both do not happen at the same time. Hence there is no restriction in taking $y^+, y^- \leq M$. Q.E.D.

Thus prepared, our main result is contained in the following

Theorem. Consider the class of goal-programming models for a single firm, specified by program (3). Denote an optimal solution by y^*, y^{+*}, y^{-*}. Then there is a vector u^* of Lagrangian multipliers, unrestricted in sign, corresponding to the goals eq. (2), so that

(5) $(p - u^*)^T y^* \geq (p - u^*)^T y$

for all $y \in Y$

y unrestricted in sign.

Proof: We shall first write down the Kuhn-Tucker theorem for problem (3), augmented with the constraints (4). Whereas the standard version

of the Kuhn-Tucker theorem is concerned with the existence of a saddle point in the whole space R^n, we shall here use a version which narrows down to the existence of a saddle point on a convex and compact set in R^n.[4)]

The following result is then obtained: The point (y^*, y^{+*}, y^{-*}) is an optimal solution to program (3) augmented with the constraints (4), if and only if there is a vector u^* unrestricted in sign so that $(y^*, y^{+*}, y^{-*}; u^*)$ is a saddle point to

$$(6) \qquad \max_{y,\, y^+,\, y^-} \; \min_{u} \quad p^T y - c^{+T} y^+ - c^{-T} y^- - u^T (y + y^+ - y^- - d)$$

subject to $\quad y \varepsilon Y$

y unrestricted in sign

$0 \leq y^+, y^- \leq M$

Eq. (5) thereupon follows from the definition of a saddle point. Q.E.D.

3. Concluding comments.

The vector u^* may be identified as a vector of fictitious excise taxes (positive entries) or subsidies (negative entries) which are used to correct the price vector p. Equation (5) states that when the total profit of the firm is calculated by using the corrected price vector, the corrected profit of the optimal production plan will exceed (or possibly be equal to) the corrected profit of any other feasible production plan.

References:

[1] Charnes, A. and W. W. Cooper: "Goal programming and multiple objective optimizations", European Journal of Operational Research, 1977, pp. 39-54.

[2] Charnes, A., W. W. Cooper and E. Rhodes: "Measuring the efficiency of decision making units", European Journal of Operational Research, 1978, pp. 429-444.

[3] Debreu, G.: Theory of Value. Wiley, New York 1959.

[4] Karlin, S.: Mathematical Methods and Theory in Games, Programming and Economics, Vol. I. McGraw-Hill, New York 1960.

[5] Ryan, M. J.: A Goal Programming Approach to Land Use Economics, Planning and Regulation. Thesis, University of Texas at Austin, 1974.

ENDNOTES

[1]See e.g. Debreu [3], Chapter 3.

[2]As is well known, these two assumptions have considerable economic substance. The convexity assumption (in conjunction with $0 \epsilon Y_j$.) rules out increasing returns to scale; it is crucial and cannot easily be relaxed. The compactness assumption is usually not included in the standard textbook formulation of a production function; it can be satisfied in most cases by suitable truncation of the production set.

[3]Cf. Charnes, Cooper and Rhodes [2].

[4]See e.g. Karlin [4], pp. 201-203. Note that Karlin makes the mistake of requiring the subset in R^n only to be convex. Thus his formulation is inadequate. If compactness is sacrificed there is no guarantee that a saddle point exists!

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